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Effects of Radiation on MHD Natural Convection Near a Vertical Plate with Oscillatory Ramped Plate Temperature

Maitree Jana
Department of Applied Mathematics
Vidyasagar University, Midnapore
721 102, India
E-mail : maitreejana@gmail.com

Sanatan Das
Department of Mathematics
University of Gour Banga,
Midnapore 732 103, India
E-mail : tutusanasd@yahoo.co.in

Rabindra Nath Jana
Department of Applied
Mathematics Vidyasagar University,
Midnapore 721 102, India
E-mail : jana261171@yahoo.co.in

Abstract **- The effects of radiation on the MHD flow past a vertical plate with oscillatory ramped plate temperature in the presence of a uniform transverse applied magnetic field have been investigated. An analytical solution of the governing equations has been obtained by employing Laplace transform technique. The numerical results for fluid velocity and temperature are presented graphically. It is found that an increase in radiation parameter leads to decrease the fluid velocity and temperature. It is also found that both the velocity as well as the temperature of the fluid decrease with an increase in Prandtl number. It is found that the shear stress due to the flow decreases with an increase in magnetic parameter while it increases with an increase in radiation parameter. Further, the rate of heat transfer at the plate increases with an increase in radiation parameter.**

Keywords **- MHD flow, natural convection, radiation parameter, Prandtl number, Grashof number and oscillatory ramped plate temperature.**

I. Introduction

The effect of radiation on MHD flow problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiative heat transfer plays significant role in the design of equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering processes. At high operating temperature, the radiation effect can be quite significant. The study of MHD flow with radiative heat transfer also plays an important role in biological Sciences. Effects of various parameters on human body can be studied and appropriate suggestions can be given to the persons working in hazardous areas having noticeable effects of magnetism and heat variation. The radiation effects on MHD free convection flow of a radiating fluid past a semi-infinite vertical plate have been investigated by Takhar et al. [1]. Abd-El-Naby et al. [2] have obtained the finite difference solution of radiation effects on MHD free convection flow over a vertical plate with variable surface temperature. Raptis et al. [3] have considered axi-symmetric flow in a thermal radiation-convection magnetohydrodynamics problem. The thermal radiation interaction on an absorbing emitting fluid past a vertical porous plate immersed in a porous medium has been studied by Samad and Rahman [4]. Mbeledogu et al. [5] have presented the unsteady MHD free convective flow of a compressible fluid past a moving vertical plate in the presence of radiative heat transfer. Ahmed and Sarmah [6] have analyzed the thermal radiation effect on a transient MHD flow with mass transfer past an impulsively started infinite vertical plate. Sangapatnam et al. [7] have studied the radiation and mass transfer effects on MHD free convective flow past an impulsively started isothermal vertical plate with dissipation. The unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion has been studied by Deka and Neogi [8]. Suneetha et al. [9] have studied the thermal radiation effects on MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. Rajesh and Vijaya Kumar Varma [10] have studied the radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. Rajesh [11] has studied radiation effects on MHD free convection flow near a vertical plate with ramped wall temperature. Anwerbeg and Ghosh [12] have investigated hydromagnetic free and forced convection of an optically-thin gray gas from vertical flat plate subject to a surface temperature oscillation with significant thermal radiation. Reddy et al. [13] have investigated unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical porous plate with variable viscosity and thermal conductivity. Rajput and Kumar [14] have studied the radiation effects on MHD flow through porous media past an exponentially accelerated vertical plate with variable temperature. Krishna and Sujatha [15] have studied MHD free and forced convective flow of Newtonian fluid through a porous medium in an infinite vertical plate in the presence of thermal radiative heat transfer and surface temperature oscillation. The radiation effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature in the presence of heat generation have been studied by Vijaya Kumar and Vijaya Kumar Varma [16]. Deka and Deka [17] have discussed the radiation effects on MHD flows past an infinite vertical plate with variable temperature and uniform mass diffusion. The radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer have been analyzed by Rajput and Kumar [18].

The aim of our present paper is to study the effects of radiation on unsteady MHD flow of a viscous incompressible radiative fluid past a vertical plate with oscillatory ramped plate temperature in the presence of a uniform transverse applied magnetic field. Initially ($t \le 0$), both the fluid and plate are at rest with constant temperature T_∞. At time $t > 0$, the plate at $z = 0$ starts to

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move in its own plane with a velocity u_0 and the temperature of the plate is raised or lowered to $T_w + (T_w - T_\infty)\cos\omega t$ when $0 < t \le t_0$ and the constant temperature T_w is maintained when $t > t_0$. An analytical solution of the governing equations have been obtained by using Laplace transformation technique. It is observed that the fluid velocity u_1 decreases with an increase in magnetic parameter M^2. The velocity u_1 decrease with an increase in radiation parameter R. The fluid velocity u_1 increases near the plate and it has oscillatory in nature away from the plate with an increase in time τ. The fluid temperature θ decreases with an increase in radiation parameter R and it increases with an increase in Prandtl number Pr. The fluid temperature θ increases near the plate and it decreases away from the plate with an increase in time τ. The shear stress τ_x due to the primary flow at the plate $(\eta = 0)$ decreases with an increase in magnetic parameter M^2. On the other hand, it is observed that the shear stress τ_x decreases with an increase in radiation parameter R. Further, the rate of heat transfer $-\theta'(0)$ at the plate $(\eta = 0)$ increases with an increase in radiation parameter R while it decreases with an increase in Prandtl number Pr.

II. Formulation of the Problem and Its Solutions

Consider the unsteady MHD flow of a viscous incompressible radiative fluid past a vertical plate. Choose a cartesian co-ordinates system with the x - axis along the plate in the vertically upward direction, the y - axis perpendicular to the plate [see Fig.1]. Initially, at time $t \le 0$, the plate and the fluid are assumed to be at the same temperature T_∞ and stationary. At time $t > 0$, the plate at $y = 0$ starts to move in its own plane with a uniform velocity u_0 and the temperature of the plate is raised or lowered to $T_w + (T_w - T_\infty)\cos\omega t$ when $0 < t \le t_0$ and the uniform temperature T_w is maintained when $t > t_0$. A uniform magnetic field B_0 is applied perpendicular to the plate. It is also assumed that the radiative heat flux in the x - direction is negligible as compared to that in the y - direction. As the plate are infinite long, the velocity and temperature fields are functions of y and t only. Since magnetic Reynolds number is very small for metallic liquid or partially ionized fluid the induced magnetic field produced by the electrically conducting fluid is negligible.

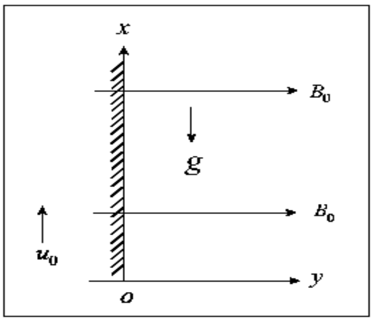

Fig.1. Geometry of the problem

The unsteady free convection flow of a radiating fluid, under usual Boussinesq approximation, is governed by the following equations

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho} u, \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}, \quad (2)$$

where u is the fluid velocity in the x-direction, T the temperature of the fluid, g the acceleration due to gravity, β the coefficient of thermal expansion, ν the kinematic coefficient of viscosity, ρ the fluid density, k the thermal conductivity, c_p the specific heat at constant pressure and q_r the radiative heat flux. The heat due to viscous dissipation is neglected for small velocities in the energy equation (2).

The initial and boundary conditions are

$t \le 0 : u = 0, T = T_\infty$ for all y

$t > 0 : u = u_0,$

$$T = \begin{cases} T_\infty + (T_\omega - T_\infty)\cos\omega t & \text{for } 0 < t < t_0 \\ T_\omega & \text{for } t > t_0 \end{cases} \quad \text{at } y = 0,$$

$$t > 0 : u \to 0, T \to T_\infty \text{ as } y \to \infty. \quad (3)$$

The radiative heat flux can be found from Rosseland approximation and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \quad (4)$$

where σ is the Stefan-Boltzman constant and k^* the spectral mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then the equation (4) can be linearized by expanding T^4 into the Taylor series about T_∞ and neglecting higher order terms to give

$$T^4 = 4T_\infty^3 T - 3T_\infty^4. \quad (5)$$

On the use of (4) and (5), the equation (2) becomes

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \quad (6)$$

Introducing dimensionless variables

$$\eta=\frac{yu_0}{\nu},\ \tau=\frac{t}{t_0},u_1=\frac{u}{u_0},\theta=\frac{T-T_\infty}{T_\omega-T_\infty},t_0=\frac{\nu}{u_0^2},\qquad(7)$$

equations (1) and (6) become

$$\frac{\partial u_1}{\partial\tau}=\frac{\partial^2u_1}{\partial\eta^2}+Gr\theta-M^2u_1,\qquad(8)$$

$$\alpha\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\eta^2},\qquad(9)$$

where $M^2=\frac{\sigma B_0^2\nu}{\rho u_0^2}$ is the magnetic parameter, $R=\frac{kk^*}{4\sigma T_\infty^3}$ the radiation parameter, $Pr=\frac{\rho\nu c_p}{k}$ the Prandtl number, $Gr=\frac{g\beta(T_\omega-T_\infty)\nu}{u_0^3}$ the Grashof number and $\alpha=\frac{3PrR}{3R+4}$.

The corresponding initial and the boundary conditions are

$$\begin{aligned}&\tau\le 0:u_1=0,\theta=0\ \text{ for all }\ \eta\\&\tau>0:u_1=1,\\&\theta=\begin{cases}\cos n\tau & \text{for }\ 0<\tau<1\\ 1 & \text{for }\ \tau>1\end{cases}\ \text{at}\ \eta=0,\\&\tau>0:u_1\to0,\theta\to0\ \text{ as }\ \eta\to\infty,\end{aligned}\qquad(10)$$

where $n=\frac{\omega\nu}{u_0^2}$ is the frequency parameter.

On the use of Laplace transformation, equations (9) and (8) become

$$\alpha\, s\,\bar\theta=\frac{d^2\bar\theta}{d\eta^2},\qquad(11)$$

$$(s+M^2)\bar u_1=\frac{d^2\bar u_1}{d\eta^2}+Gr\bar\theta,\qquad(12)$$

where

$$\begin{aligned}\bar u_1(\eta,s)&=\int_0^\infty u_1(\eta,\tau)e^{-s\tau}\,d\tau,\\ \bar\theta(\eta,s)&=\int_0^\infty\theta(\eta,\tau)e^{-s\tau}\,d\tau.\end{aligned}\qquad(13)$$

The corresponding boundary conditions for $\bar u_1$ and $\bar\theta$ are

$$\begin{aligned}&\bar u_1=\frac{1}{s},\ \bar\theta=\lambda\ \text{ at }\ \eta=0,\\&\bar u_1\to0,\ \bar\theta\to0\ \text{ as }\ \eta\to\infty,\end{aligned}\qquad(14)$$

where

$$\lambda=\frac{1}{2}\left[\left(\frac{1}{s-in}+\frac{1}{s+in}\right)-\left\{\frac{e^{-(s-in)}}{(s-in)}+\frac{e^{-(s+in)}}{(s+in)}\right\}\right]+\frac{e^{-s}}{s}\qquad(15)$$

The solution of the equations (11) and (12) subject to the boundary conditions (14) are easily obtained and are given by

$$\bar\theta(\eta,s)=\begin{cases}\lambda e^{-\sqrt{\alpha s}\,\eta} & \text{for }\alpha\ne1,\\ \lambda e^{-\sqrt{s}\,\eta} & \text{for }\alpha=1,\end{cases}\qquad(16)$$

$$\bar u_1(\eta,s)=\begin{cases}\dfrac{e^{-\sqrt{s+M^2}\,\eta}}{s}+\dfrac{\lambda Gr}{(s-b)(\alpha-1)}\left[e^{-\sqrt{s+M^2}\,\eta}-e^{-\sqrt{s\alpha}\,\eta}\right] & \text{for }\alpha\ne1,\\[2ex] \dfrac{e^{-\sqrt{s+M^2}\,\eta}}{s}+\dfrac{\lambda Gr}{M^2}\left[e^{-\sqrt{s}\,\eta}-e^{-\sqrt{s+M^2}\,\eta}\right] & \text{for }\alpha=1,\end{cases}\qquad(17)$$

where λ is given by (15).

The inverse transforms of equations (16) and (17) give for the temperature distribution and velocity fields as

$$\theta(\eta,\tau)=\begin{cases}\frac{1}{2}\Big[F_2(\eta,\alpha,in,\tau)+F_2(\eta,\alpha,-in,\tau)\\ -H(\tau-1)\big\{e^{in}F_2(\eta,\alpha,in,\tau-1)\\ +e^{-in}F_2(\eta,\alpha,-in,\tau-1)-2F_1(\eta,\alpha,\tau-1)\big\}\Big] & \text{for }\ \alpha\ne1,\\[2ex] \frac{1}{2}\Big[F_{11}(\eta,in,\tau)+F_{11}(\eta,-in,\tau)\\ -H(\tau-1)\big\{e^{in}F_{11}(\eta,in,\tau-1)\\ +e^{-in}F_{11}(\eta,-in,\tau-1)-2F_{10}(\eta,\tau-1)\big\}\Big] & \text{for }\ \alpha=1,\end{cases}\qquad(18)$$

$$F(\eta,\tau)=\begin{cases} F_3(\eta,M^2,\tau)+\dfrac{Gr}{2(\alpha-1)}\Big[F_5(\eta,M^2,b,in,\tau)+F_5(\eta,M^2,b,-in,\tau) \\ -F_7(\eta,\alpha,b,in,\tau)-F_7(\eta,\alpha,b,-in,\tau) \\ -H(\tau-1)\{e^{in}F_5(\eta,M^2,b,in,\tau-1)+e^{-in}F_5(\eta,M^2,b,-in,\tau-1) \\ -e^{in}F_7(\eta,\alpha,b,in,\tau-1)-e^{-in}F_7(\eta,\alpha,b,-in,\tau-1)\} \\ +F_8(\eta,M^2,b,\tau-1)-F_9(\eta,\alpha,b,\tau-1)\Big] & \text{for } \alpha\neq 1, \\ \\ F_3(\eta,M^2,\tau)-\dfrac{Gr}{2M^2}\Big[F_4(\eta,M^2,in,\tau)+F_4(\eta,M^2,-in,\tau) \\ -F_{11}(\eta,in,\tau)-F_{11}(\eta,-in,\tau) \\ -H(\tau-1)\{e^{in}F_4(\eta,M^2,in,\tau-1)+e^{-in}F_4(\eta,M^2,-in,\tau-1) \\ -e^{in}F_{11}(\eta,in,\tau-1)-e^{-in}F_{11}(\eta,-in,\tau-1) \\ -F_3(\eta,M^2,\tau-1)+F_{10}(\eta,\tau-1)\}\Big] & \text{for } \alpha=1, \end{cases} \quad (19)$$

where

$$F_1(\eta,\alpha,\tau)=\text{erfc}\left(\frac{\eta}{2}\sqrt{\frac{\alpha}{\tau}}\right),$$

$$F_2(\eta,\alpha,x,\tau)=\frac{1}{2}e^{x\tau}\left[e^{\eta\sqrt{\alpha x}}\ \text{erfc}\left(\frac{\eta}{2}\sqrt{\frac{\alpha}{\tau}}+\sqrt{x\tau}\right)+e^{-\eta\sqrt{\alpha x}}\ \text{erfc}\left(\frac{\eta}{2}\sqrt{\frac{\alpha}{\tau}}-\sqrt{x\tau}\right)\right],$$

$$F_3(\eta,M^2,\tau)=\frac{1}{2}\left[e^{M\eta}\ \text{erfc}\left(\frac{\eta}{2\sqrt{\tau}}+M\sqrt{\tau}\right)+e^{-M\eta}\ \text{erfc}\left(\frac{\eta}{2\sqrt{\tau}}-M\sqrt{\tau}\right)\right],$$

$$F_4(\eta,x,M^2,\tau)=\frac{1}{2}e^{x\tau}\left[e^{\eta\sqrt{M^2+x}}\ \text{erfc}\left\{\frac{\eta}{2\sqrt{\tau}}+\sqrt{(M^2+x)\tau}\right\}\right.$$
$$\left.+e^{-\eta\sqrt{M^2+x}}\ \text{erfc}\left\{\frac{\eta}{2\sqrt{\tau}}-\sqrt{(M^2+x)\tau}\right\}\right],$$

$$F_5(\eta,M^2,b,x,\tau)=\frac{1}{(b-x)}\left[F_4(\eta,M^2,b,\tau)-F_4(\eta,M^2,x,\tau)\right], \quad (20)$$

$$F_6(\eta,\alpha,b,\tau)=\frac{1}{2}e^{b\tau}\left[e^{\sqrt{\alpha b}\,\eta}\ \text{erfc}\left(\frac{\eta}{2}\sqrt{\frac{\alpha}{\tau}}+\sqrt{\alpha b\tau}\right)+e^{-\sqrt{\alpha b}\,\eta}\ \text{erfc}\left(\frac{\eta}{2}\sqrt{\frac{\alpha}{\tau}}-\sqrt{\alpha b\tau}\right)\right],$$

$$F_7(\eta,\alpha,b,x,\tau)=\frac{1}{(b-x)}\left[F_6(\eta,\alpha,b,\tau)-F_2(\eta,\alpha,x,\tau)\right],$$

$$F_8(\eta,M^2,b,\tau-1)=\frac{1}{b}\left[F_4(\eta,M^2,b,\tau-1)-F_3(\eta,M^2,\tau-1)\right],$$

$$F_9(\eta,\alpha,b,\tau-1)=\frac{1}{b}\left[F_6(\eta,\alpha,b,\tau-1)-F_1(\eta,\alpha,\tau-1)\right],$$

$$F_{10}(\eta,\tau)=\text{erfc}\left(\frac{\eta}{2\sqrt{\tau}}\right),$$

$$F_{11}(\eta,x,\tau)=\frac{1}{2}e^{x\tau}\left[e^{\eta\sqrt{x}}\ \text{erfc}\left(\frac{\eta}{2\sqrt{\tau}}+\sqrt{x\tau}\right)+e^{-\eta\sqrt{x}}\ \text{erfc}\left(\frac{\eta}{2\sqrt{\tau}}-\sqrt{x\tau}\right)\right],$$

and $b=\dfrac{M^2}{\alpha-1}$ and x is dummy variable and F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10} and F_{11} are the dummy functions and $\text{erfc}(.)$ being the complementary error function.

III. Results and Discussion

We have presented the non-dimensional fluid velocity u_1 and the fluid temperature θ for several values of the magnetic parameter M^2, radiation parameter R, Grashof number Gr, Prandtl number Pr, frequency parameter n and time τ in Figs.2-11. It is seen from Fig.2 that the velocity u_1 decreases with an increase in magnetic parameter M^2. This implies that the magnetic field has retarding influence on the velocity field. It is revealed from Fig.3 that the velocity u_1 decreases with an increase in radiation parameter R. This means that the radiation decelerates the fluid velocity. Fig.4 shows that the primary velocity u_1 increases with an increase in Grashof number Gr. Increasing the value of Gr have the tendency to induce more flow and hence the fluid velocity accelerates. Fig.5 displays that the velocity u_1 decreases with an increase in Prandtl number Pr. This is due to the fact that the fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly. It is seen from Fig.6 that the velocity u_1 increases with an increase in frequency parameter n. Fig.7 reveals that the velocity u_1 increases near the plate and it oscillates away from the plate with an increase in time τ. It is observed from Fig.8 that the fluid temperature θ decreases with an increase in radiation parameter R. In the presence of radiation, the thermal boundary layer always found to thicken which implies that the radiation provides an additional means to diffuse energy. This means that the thermal boundary layer decreases and more uniform temperature distribution across the boundary layer. Fig.9 shows that the fluid temperature θ decreases with an increase in Prandtl number Pr. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Pr. Hence in the case of smaller Prandtl numbers as thermal boundary layer is thicker and the heat transfer is reduced. It is seen from Fig.10 that the fluid temperature θ increases with an increase in frequency parameter n. Fig.11 displays that the fluid temperature θ increases near the plate and it decreases away from the plate with an increase in time τ.

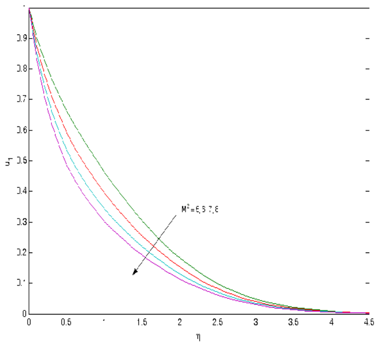

Fig.2. Velocity u_1 for different M^2 when $R = 2$, $n = 2$, $Pr = 0.71$, $Gr = 5$ and $\tau = 0.5$

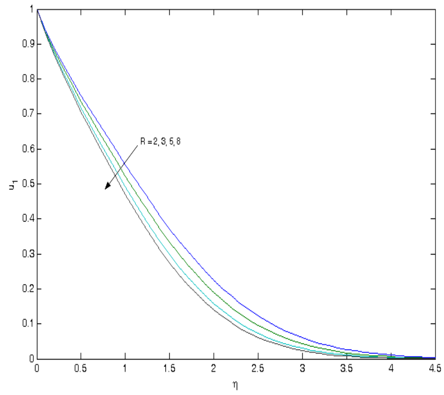

Fig.3. Velocity u_1 for different R when $M^2 = 4$, $n = 2$, $Pr = 0.71$, $Gr = 5$ and $\tau = 0.5$

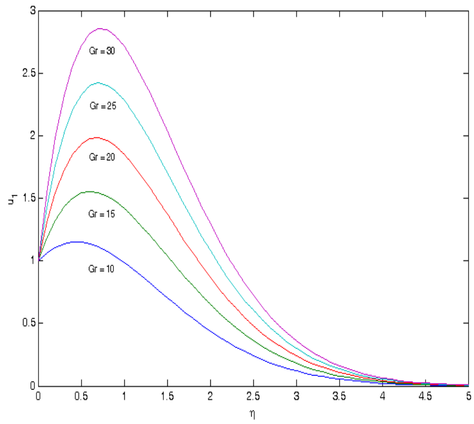

Fig.4. Velocity u_1 for different Gr when $M^2 = 4$, $R = 2$, $n = 2$, $Pr = 0.71$ and $\tau = 0.5$

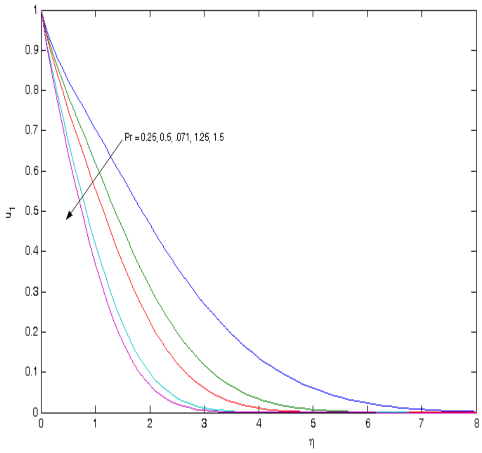

Fig.5. Velocity u_1 for different Pr when $M^2 = 4$, $R = 2$, $n = 2$, $Gr = 5$ and $\tau = 0.5$

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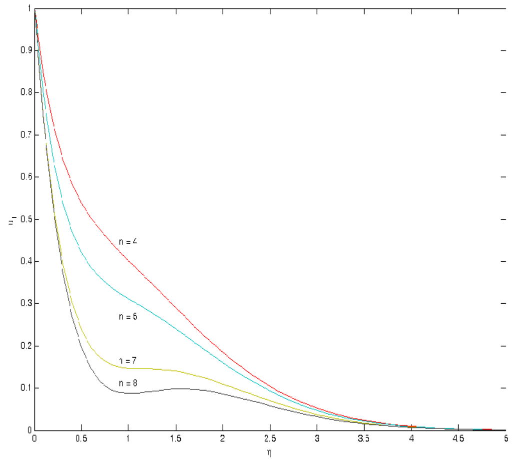

Fig.6. Velocity u_1 for different n when $M^2 = 4$, $R = 2$, $Pr = 0.71$, $Gr = 5$ and $\tau = 0.5$

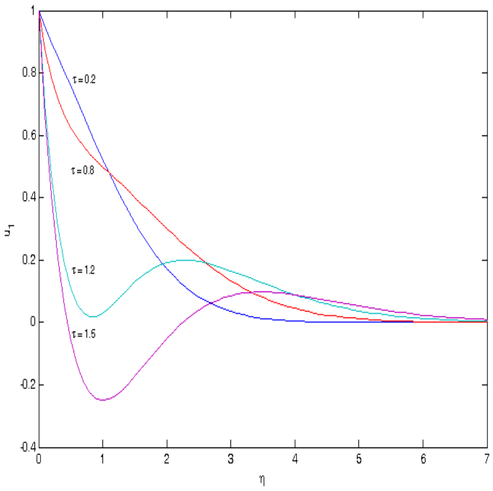

Fig.7. Velocity u_1 for different τ when $R = 2$, $n = 2$, $Pr = 0.71$ and $Gr = 5$

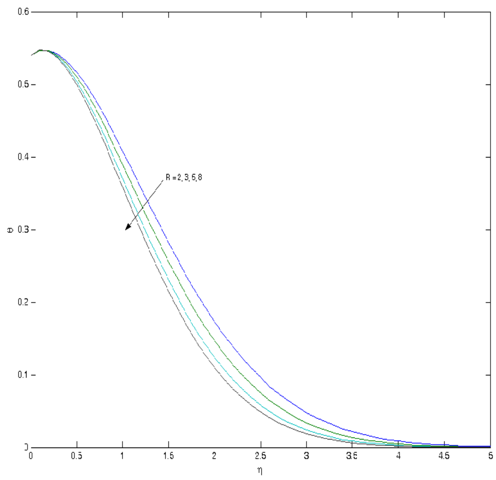

Fig.8. Temperature for different R when $n = 2$, $Pr = 0.71$ and $\tau = 0.5$

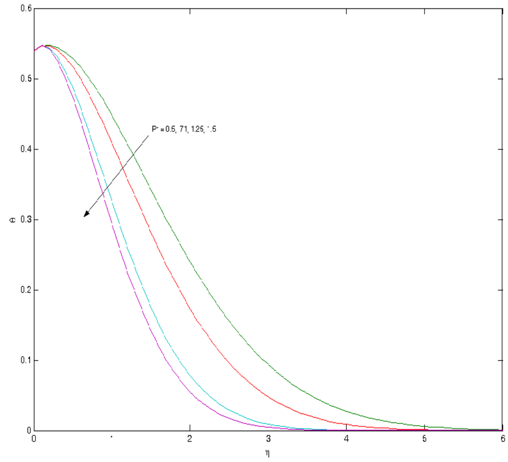

Fig.9. Temperature for different Pr when $n = 2$, $R = 2$ and $\tau = 0.5$

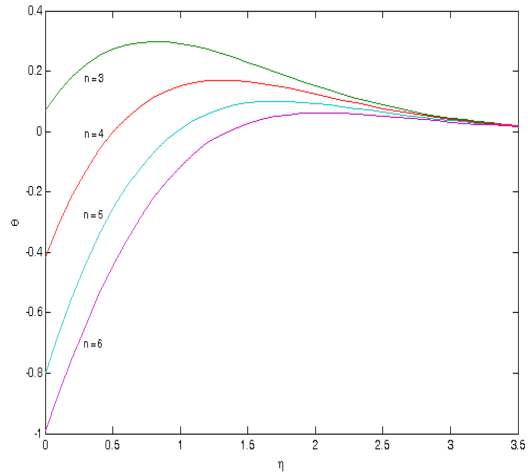

Fig.10. Temperature for different n when $R = 2$, $Pr = 0.71$ and $\tau = 0.5$

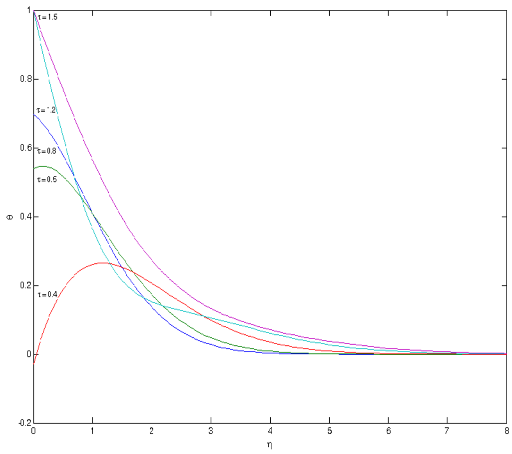

Fig.11. Temperature for different τ when $n = 2$, $Pr = 0.71$ and $R = 2$

The rate of heat transfer and the non-dimensional shear stresses due the primary and the secondary flows at the plate $\eta = 0$ are given by

$$\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}=\begin{cases}-\frac{1}{2}\Big[H_2(\alpha,in,\tau)+H_2(\alpha,-in,\tau)\\ -H(\tau-1)\Big\{e^{in}H_2(\alpha,in,\tau-1)\\ +e^{-in}H_2(\alpha,-in,\tau-1)-2H_1(\alpha,\tau-1)\Big\}\Big] & \text{for } \alpha\neq 1,\\ \\ -\frac{1}{2}\Big[H_{11}(in,\tau)+H_{11}(-in,\tau)\\ -H(\tau-1)\Big\{e^{in}H_{11}(in,\tau-1)\\ +e^{-in}H_{11}(-in,\tau-1)-2H_{10}(\tau-1)\Big\}\Big] & \text{for } \alpha=1,\end{cases} \tag{21}$$

$$\tau_x+i\tau_y=\left(\frac{\partial F}{\partial\eta}\right)_{\eta=0}$$

$$=\begin{cases}-H_3(M^2,\tau)-\frac{Gr}{2(\alpha-1)}\Big[H_5(M^2,b,in,\tau)+H_5(M^2,b,-in,\tau)\\ -H_7(\alpha,b,in,\tau)-H_7(\alpha,b,-in,\tau)\\ -H(\tau-1)\{e^{in}H_5(M^2,b,in,\tau-1)+e^{-in}H_5(M^2,b,-in,\tau-1)\\ -e^{in}H_7(\alpha,b,in,\tau-1)-e^{-in}H_7(\alpha,b,-in,\tau-1)\}\\ -2H_8(M^2,b,\tau-1)+2H_9(\alpha,b,\tau-1)\Big] & \text{for } \alpha\neq 1,\\ \\ -H_3(M^2,\tau)+\frac{Gr}{2M^2}\Big[H_4(M^2,in,\tau)+H_4(M^2,-in,\tau)\\ -H_{11}(in,\tau)-H_{11}(-in,\tau)\\ -H(\tau-1)\{e^{in}H_4(M^2,in,\tau-1)+e^{-in}H_4(M^2,-in,\tau-1)\\ -e^{in}H_{11}(in,\tau-1)-e^{-in}H_{11}(-in,\tau-1)\}\\ -2H_3(M^2,\tau-1)+2H_{10}(\tau-1)\Big] & \text{for } \alpha=1,\end{cases} \tag{22}$$

where

$$\begin{aligned}
H_1(\alpha,\tau)&=\sqrt{\frac{\alpha}{\pi\tau}},\\
H_2(\alpha,x,\tau)&=e^{x\tau}\left[\sqrt{\alpha x}\ \mathrm{erf}\left(\sqrt{x\tau}\right)+\sqrt{\frac{\alpha}{\pi\tau}}\ e^{-x\tau}\right],\\
H_3(M^2,\tau)&=M\ \mathrm{erf}\left(M\sqrt{\tau}\right)+\frac{1}{\sqrt{\pi\tau}}\ e^{-M^2\tau},\\
H_4(x,M^2,\tau)&=e^{x\tau}\left[\sqrt{M^2+x}\ \mathrm{erf}\left\{\sqrt{(M^2+x)\tau}\right\}+\frac{1}{\sqrt{\pi\tau}}\ e^{-(M^2+x)\tau}\right],\\
H_5(M^2,b,x,\tau)&=\frac{1}{(b-x)}\left[H_4(M^2,b,\tau)-H_4(M^2,x,\tau)\right],\\
H_6(\alpha,b,\tau)&=e^{b\tau}\left[\sqrt{\alpha\,b}\ \mathrm{erf}\left(\sqrt{\alpha b\tau}\right)+\sqrt{\frac{\alpha}{\pi\tau}}\ e^{-\alpha b\tau}\right],\\
H_7(\alpha,b,x,\tau)&=\frac{1}{(b-x)}\left[H_6(\alpha,b,\tau)-H_2(\alpha,x,\tau)\right],\\
H_8(M^2,b,\tau-1)&=\frac{1}{b}\left[H_4(M^2,b,\tau-1)-H_3(M^2,\tau-1)\right],
\end{aligned} \tag{23}$$

$$H_9(\alpha,b,\tau-1)=\frac{1}{b}\left[H_6(\alpha,b,\tau-1)-H_1(\alpha,\tau-1)\right],$$

$$H_{10}(\tau)=\frac{1}{\sqrt{\pi\tau}},$$

$$H_{11}(x,\tau)=e^{x\tau}\left[\sqrt{x}\ \mathrm{erf}\left(\sqrt{x\tau}\right)+\frac{1}{\sqrt{\pi\tau}}e^{-x\tau}\right],$$

where H_1, H_2, H_3, H_4, H_5, H_7, H_8, H_9, H_{10} and H_{11} are dummy functions.

Numerical results of the rate of heat transfer $-\theta'(0)$ at the plate $(\eta=0)$ are presented in the Table 1 for several values of Prandtl number Pr and time τ against the radiation parameter R. Table 1 shows that for the fixed value of radiation parameter R, the rate of heat transfer $-\theta'(0)$ decreases with an increase in Prandtl number Pr. This may be explained by the fact that frictional forces become dominant with increasing values of Pr and hence yield greater heat transfer rates. It is also seen that the rate of heat transfer $-\theta'(0)$ decreases for $\tau\le 1$ and it increases for $\tau>1$ for the fixed value of radiation parameter R. Further, it is observed that for fixed value of Pr and τ, the rate of heat transfer $-\theta'(0)$ increases with an increase in radiation parameter R.

Table 1. Rate of heat transfer $-\theta'(0)$ at the plate $\eta=0$ when $n=2$

	Pr				τ			
R	0.25	0.5	0.71	1.5	0.3	0.4	0.5	1.5
2	0.62300	0.50794	0.48007	0.24927	1.04846	0.74483	0.43279	1.44773
3	0.81808	0.71182	0.63536	0.43067	1.28881	0.97402	0.63536	1.74575
4	0.97773	0.88106	0.80737	0.59432	1.50488	1.17443	0.80737	2.00580
5	1.11570	1.02718	0.95743	0.74322	1.70139	1.35296	0.95743	2.23858

Numerical values of the non-dimensional shear stress τ_x due to the flow at the plate $(\eta=0)$ are presented in Figs.12-16 for several values of radiation parameter R, Grashof number Gr, Prandtl number Pr, frequency parameter n and time τ against magnetic parameter M^2. Fig.12 shows that for the fixed values of the magnetic parameter M^2, the shear stress τ_x increases with an increase in radiation parameter R. On the other hand, it is observed that the shear stress τ_x decreases with an increase in magnetic parameter M^2 for the fixed values of the radiation parameter R. Fig.13 reveals that the shear stress τ_x increases with an increase in Grashof number Gr. Fig.14 shows that the shear stress τ_x increases with an increase in Prandtl number Pr. It is illustrated from Fig.15 that the shear stress τ_x decreases with an increase in frequency parameter n. Fig.16 displays that the shear stresses τ_x decreases with an increase in time τ.

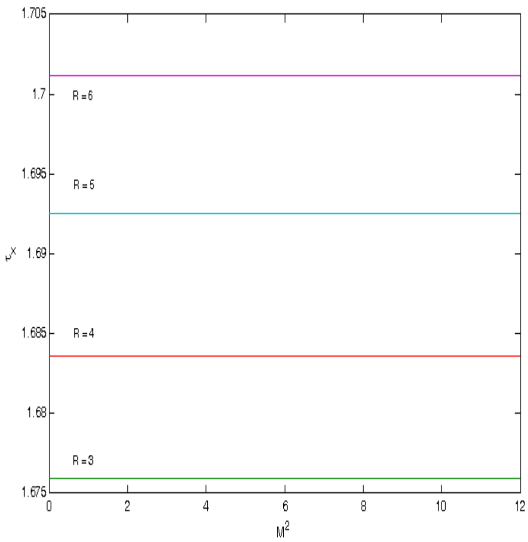

Fig.12. Shear stress τ_x for different $\boldsymbol{R}$ when $\boldsymbol{n=2}$, $\boldsymbol{Pr=0.71}$, $\boldsymbol{Gr=5}$ and $\boldsymbol{\tau=0.5}$

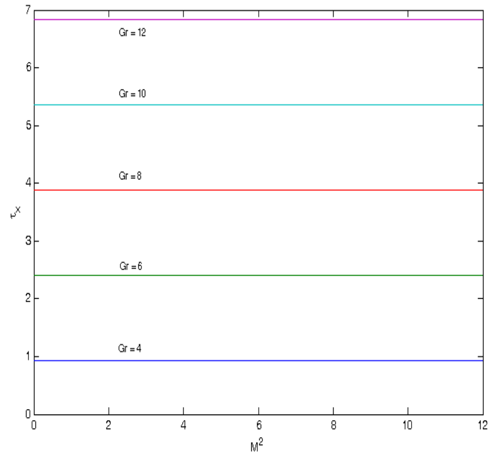

Fig.13. Shear stress τ_x for different $\boldsymbol{Gr}$ when $\boldsymbol{R=2}$, $\boldsymbol{n=2}$, $\boldsymbol{Pr=0.71}$ and $\boldsymbol{\tau=0.5}$

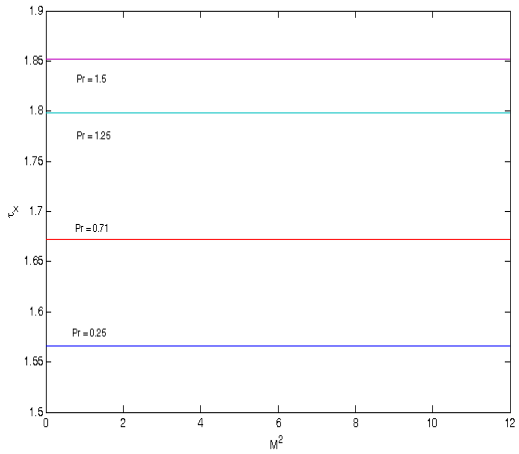

Fig.14. Shear stress τ_x for different $\boldsymbol{Pr}$ when $\boldsymbol{R=2}$, $\boldsymbol{n=2}$, $\boldsymbol{Gr=5}$ and $\boldsymbol{\tau=0.5}$

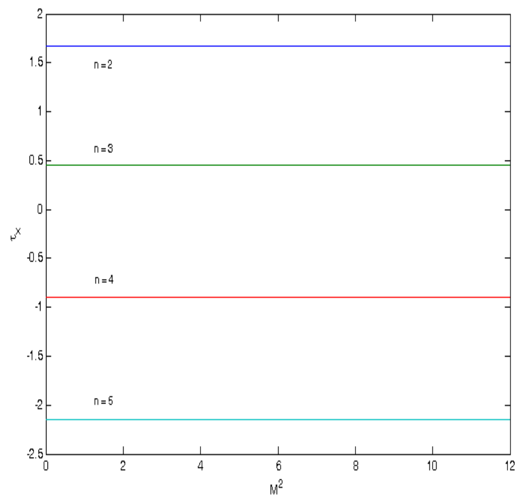

Fig.15. Shear stress τ_x for different $\boldsymbol{n}$ when $\boldsymbol{R=2}$ $\boldsymbol{Pr=0.71}$, $\boldsymbol{Gr=5}$ and $\boldsymbol{\tau=0.5}$

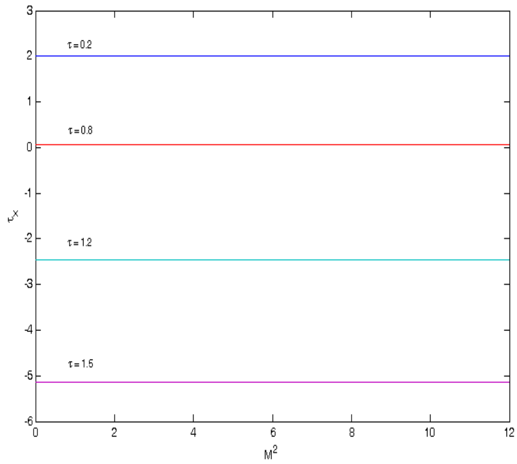

Fig.16. Shear stress τ_x for different τ when $\boldsymbol{R=2}$, $\boldsymbol{n=2}$, $\boldsymbol{Pr=0.71}$ and $\boldsymbol{Gr=5}$

IV. Conclusion

The effects of radiation on the MHD flow past a vertical plate with oscillatory ramped plate temperature in the presence of a uniform applied transvrse magnetic field have been studied. It is found that an increase in radiation parameter leads to decrease the fluid velocity and temperature. It is also found that both the velocity as well as the temperature of the fluid decrease with an increase in Prandtl number. Further, it is observed that the velocity increases with an increase in Grashof number and an increase in time leads to increase the fluid velocity and temperature.

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AUTHOR’S PROFILE

Miss Maitree Jana

is a research scholar . She had her B.Sc.(Honours) from Midnapore College (2004) and M.Sc. in Applied Mathematics from Vidyasagar University (2006). Her field of research is fluid dynamics with or without magnetic field.

Mr. Sanatan Das

is an Assistant Professor of the department of Mathematics, University of Gour Banga, Malda 732 103, India. He was formerly a lecturer of the department of Mathematics, Islampur College, Islampur 733 202, India. He had his B.Sc. (Honours) from Midnapore College (1997) and M.Sc. in Applied Mathematics from Vidyasagar University (1999). His areas of interest include fluid dynamics, magnetohydrodynamics, heat transfer and porous media. He has to his credit 50 research papers in journals of national and international repute and he is the author of several books. Mr. Das has been undertaking various administrative work of the university at different positions.

Prof. Rabindra Nath Jana

is a Professor of the department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India. He was formerly a Lecturer of the department of Mathematics, Kharagpur College, Kharagpur. He had his B.Sc.(Honours) from Kharagpur College (1969), M.Sc. in Applied Mathematics from IIT Kharagpur (1971), DIIT from IIT Kharagpur (1972) and Ph.D. from IIT Kharagpur (1976) . His areas of interest include fluid dynamics, magnetohydrodynamics, heat transfer and porous media. He has to his credit 108 research papers in journals of national and international repute and he is the author of several books. Prof. Jana has been undertaking various administrative work of the university at different positions.